

Déterminer les *conditions d'existence* et calculer les expressions suivantes (en pensant à *simplifier* le cas échéant) :

$$(a) \ A(x) = \frac{3}{x-6} - \frac{1}{x-2}$$

$$(b) \ B(x) = \frac{x+5}{x-5} - \frac{x-5}{x+5}$$

$$(c) \ C(x) = \frac{3x}{x^2-3x} + \frac{2x}{x^2-9}$$

$$(d) \ D(x) = \frac{x}{5-2x} + \frac{5-x}{2x-5}$$

$$(e) \ E(x) = \frac{2x-4}{3x^3-6x^2} + \frac{1}{2x^4-2x^3}$$

$$(f) \ F(x) = \frac{1}{x-1} + \frac{1}{x^2-3x+2} + \frac{1}{x^2-5x+6}$$

$$(g) \ G(x) = \frac{2x^2+4x}{x^2+3x+2} + \frac{3x^2+1}{1-x^2}$$

$$(h) \ H(x) = \frac{x}{1-2x} - \frac{x+2}{1+2x} + \frac{4x^2}{4x^2-1}$$

$$(i) \ I(x) = \frac{x+1}{2x-2} - \frac{x-1}{2x+2} - \frac{4x}{x^2-1} - \frac{x^2+1}{1-x^2}$$

$$(j) \ J(x) = \frac{1-x}{3x-2} - \frac{3x}{2+3x} - \frac{12x^2}{4-9x^2}$$

$$(k) \ K(x) = \frac{3x}{3x^2-x} - \frac{6x-4}{9x^2-6x+1} + \frac{7x+9}{(5x+4)^2-(2x+5)^2}$$

$$(l) \ L(x) = \frac{3+x}{x^2-9} - \frac{x^2-6x+9}{3x-x^2} - \frac{x^2-1}{x^2-4x+3}$$

$$(m) \ M(x) = \frac{1+2x}{4x^2-1} - \frac{4x^2-4x+1}{2x^2-x} + \frac{5x^2-2x}{x^2-2x^3}$$

$$(n) \ N(x) = \frac{1-9x^2}{3x^3-2x^2-7x-2} + \frac{x^2+2}{x^4+x^2-2}$$

$$(o) \ O(x) = \frac{x^2-9}{x+2} \cdot \frac{2x+4}{x^2-6x+9}$$

$$(p) \ P(x) = \frac{15x-30}{2x} \cdot \frac{3x^2}{10-5x}$$

$$(q) \ Q(x) = \frac{x-2}{x} : \frac{4-2x}{x^3}$$

$$(r) \ R(x) = \frac{x^2+x-2}{x^2-5x+4} : \frac{x^2-x-6}{x^2+x-20}$$

$$(s) \ S(x) = \frac{1}{x - \frac{x^2-1}{x+\frac{1}{x-1}}}$$

**Solutions.** (a)  $x \in \mathbb{R} \setminus \{2; 6\} : A(x) = \frac{3}{x-6} - \frac{1}{x-2}$  (b)  $x \in \mathbb{R} \setminus \{-5; 5\} : B(x) = \frac{(x-6)(x-2)}{2x^2-6x}$  (c)  $x \in \mathbb{R} \setminus \{-3; 3\} : C(x) = \frac{(x-6)(x+5)}{6x^2-5x}$

(d)  $x \in \mathbb{R} \setminus \{0; 2\} : D(x) = \frac{1-x}{x-2}$  (e)  $x \in \mathbb{R} \setminus \{-3; 5\} : E(x) = \frac{4x^2-4x-3}{4x^2-4x+3}$  (f)  $x \in \mathbb{R} \setminus \{-1; 1\} : F(x) = \frac{\frac{3x}{x+1}-1}{x-1}$

(g)  $x \in \mathbb{R} \setminus \{0; 1; 2\} : G(x) = \frac{\frac{2x^2+4x}{x^2+3x+2} + \frac{3x^2+1}{1-x^2}}$  (h)  $x \in \mathbb{R} \setminus \{-1; 1\} : H(x) = \frac{4x^2-4x+1}{4x^2-1}$

(i)  $x \in \mathbb{R} \setminus \{-1; 1\} : I(x) = \frac{x+1}{2x-2} - \frac{x-1}{2x+2} - \frac{4x}{x^2-1} - \frac{x^2+1}{1-x^2}$  (j)  $x \in \mathbb{R} \setminus \{-1; 1\} : J(x) = \frac{1-x}{3x-2} - \frac{3x}{2+3x} - \frac{12x^2}{4-9x^2}$

(k)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : K(x) = \frac{\frac{3x}{x-1}-\frac{5}{2}}{\frac{3x^2-3x+1}{x-1}-\frac{5}{2}}$  (l)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : L(x) = \frac{\frac{3+x}{x^2-9}-\frac{x^2-6x+9}{3x-x^2}}$

(m)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : M(x) = \frac{\frac{1+2x}{4x^2-1}-\frac{4x^2-4x+1}{2x^2-x}}$  (n)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : N(x) = \frac{\frac{1-9x^2}{3x^3-2x^2-7x-2}+\frac{x^2+2}{x^4+x^2-2}}$

(o)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : O(x) = \frac{\frac{x^2-9}{x+2}\cdot\frac{2x+4}{x^2-6x+9}}$  (p)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : P(x) = \frac{\frac{15x-30}{2x}\cdot\frac{3x^2}{10-5x}}$

(q)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : Q(x) = \frac{\frac{x-2}{x}\cdot\frac{4-2x}{x^3}}$  (r)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : R(x) = \frac{\frac{x^2+x-2}{x^2-5x+4}\cdot\frac{x^2-x-6}{x^2+x-20}}$

(s)  $x \in \mathbb{R} \setminus \{-1; 1; 2\} : S(x) = \frac{1}{x-\frac{x^2-1}{x+\frac{1}{x-1}}}$